APPLICATIONS OF THE LAGRANGIAN DYNAMIC MODEL IN LES OF TURBULENT FLOW OVER SURFACES WITH HETEROGENEOUS ROUGHNESS DISTRIBUTIONS

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ABSTRACT

We study turbulent flow over surfaces with varying roughness scales, using large eddy simulation (LES). The goal is to use LES results to formulate effective boundary conditions in terms of effective roughness height and blending height, to be used for RANS. The LES are implemented with the dynamic Smagorinsky model based on the Germano identity. However, as is well-known, when this identity is applied locally, it yields a coefficient with unphysically strong fluctuations and averaging is needed for better realism and numerical stability. The traditional approach consists of averaging over homogeneous directions, for example horizontal planes in channel flow. This requirement for homogeneous directions in the flow field and the concomitant inability to handle complex geometries renders the use of this model questionable in studying the effect of surface heterogeneity. Instead, a new version of the Lagrangian dynamic subgrid-scale (SGS) model [1] is implemented. A systematic set of simulations of flow over patches of differing roughness is performed, covering a wide range of patch length scales and surface roughness values. The simulated mean velocity profiles are analyzed to identify the height of the blending layer and used to measure the effective roughness length. Extending ideas introduced by Miyake [2] and Claussen [3], we have proposed a simple expression for effective surface roughness and blending height knowing local surface patch roughness values and their lengths [4]. Results of the model agreed well with the LES results when the heterogeneous surface consisted of patches of equal sizes. The model is tested here for surfaces with patches of different sizes.

INTRODUCTION

Flows over surfaces with abrupt changes in surface roughness and heterogeneous roughness distributions occur in a wide range of engineering applications involving turbulent flows and in the environment (Atmospheric Boundary Layer flow). Consequently, the effect of variability in surface roughness on boundary layers continues to be the subject of numerous studies [5-9].

These studies attempt to understand and quantify the flow disturbances caused by the roughness change. In addition, a parameterization of the effect of heterogeneity is often desirable; for example, numerical simulations that cannot resolve the heterogeneity scale need to model its effect on resolved scales. Analytical and experimental techniques have been traditionally used to tackle the heterogeneity problem. More recently, numerical techniques, particularly Large-Eddy Simulation, has become increasingly popular as a tool for a physical understanding of the dynamics of the blending phenomena over heterogeneous surfaces [9-15]. This paper continues this previous body of work by using LES, with a new generation model for subgrid-scale stresses, to test a new parameterization for heterogeneous surfaces at high Reynolds numbers.

LARGE EDDY SIMULATION AND THE LAGRANGIAN DYNAMIC SGS MODEL

Large Eddy Simulation assumes that the largest eddies contain most of the energy and are responsible for most of the transport of momentum and scalars. LES consists of solving the Navier-Stokes equations with eddies smaller than the filter size excluded, whereas eddies larger than the filter size can be resolved [16]. The spatial filtering of the Navier-Stokes
equations gives rise to a subgrid-scale (SGS) flux term $\sigma_{ij}$; it represents the effect of small eddies that are filtered out on the resolved scales of motion. The modeling of this term is required to close the LES system of equations, which can be written in rotational form as

$$\frac{\partial \bar{\mathbf{u}}_i}{\partial t} + \bar{u}_j \left( \frac{\partial \bar{\mathbf{u}}_i}{\partial x_j} - \frac{\partial \bar{\mathbf{u}}_j}{\partial x_i} \right) = -\frac{1}{\rho} \frac{\partial \tau_{ij}}{\partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} + \bar{F}_i, \quad \frac{\partial \bar{\mathbf{u}}_i}{\partial x_j} = 0. \quad (1)$$

where a tilde ($\tilde{\cdot}$) denotes filtering at the grid scale $\Delta$, $u_i$ is one of the three components of the velocity vector, $\tau_{ij}$ is the deviatoric part of the SGS stress tensor $\sigma_{ij}$, the symmetric part being lumped into the pressure term

$$p^* = p - \frac{1}{3} \rho \sigma_{kk} + \frac{1}{2} \rho \bar{u}_j \bar{u}_j. \quad (2)$$

Since here we consider flows with very high Reynolds number well above the viscous sublayer (not wall-resolved LES), turbulence near a wall cannot be assumed isotropic and homogeneous and hence the appropriate value $c\Delta$ may differ from 0.17.

The dynamic approach ‘measures’ the model coefficient dynamically during the simulation by using information from the smallest resolved scales [19]. The approach uses the assumption of scale-invariance, $c_{\Delta_2} = c_{\Delta_1}$ by applying the coefficient measured from the resolved scales to the subgrid-scale range [20]. However, scale-invariance near the grid-filter scale $\Delta$ does not always hold. The coefficient depends strongly on scale when $\Delta$ tends to the integral scale (as occurs in the first few grid-points near the ground in LES of wall bounded flows that does not resolve the viscous sublayer). To account for scale effects, scale-dependent formulations have been successfully implemented for atmospheric boundary layer (ABL) flows by Porté-Agel et al. [21]. A power-law behavior is assumed for the scale dependence of $c_\Delta$: $c_{\Delta_2} \sim c_{\Delta_1} \Delta_2 / \Delta_1$, and the case $\Phi=0$ (i.e. $\beta=1$) corresponds to the scale-invariant standard dynamic model.

In Porté-Agel et al. [21], the scaling coefficient $\beta$ itself was obtained dynamically using an additional filtering operation at a scale $4\Delta$. In this work, for computational simplicity a dynamic computation of $\beta$ is avoided. Instead, the vertical profile of $\beta$ as a function of $z/\Delta$ is taken from Porté-Agel et al. [21] and imposed as

$$\beta = 1 - 0.65 e^{-0.7z/\Delta}, \quad \zeta \text{ being the distance from the surface.}$$

$\beta$ being the distance from the surface. The dynamic model, even its scale-dependent version, yields a coefficient with strong fluctuations and averaging is required for numerical stability. The standard approach [19] averages over homogeneous directions, for example horizontal planes in wall-bounded flows. This requirement for homogeneous directions in the flow field and the concomitant inability to handle complex geometries renders the use of this model questionable in studying the effect of surface heterogeneity. Instead, a new version of the so-called Lagrangian dynamic SGS model [1] is used here.

In this approach, averages are obtained in time following fluid pathlines rather than over homogeneous directions. The Lagrangian model is very well suited for applications with heterogeneous spatial conditions since it preserves local variability, preserves Galilean invariance, and does not require homogeneous directions. The model starts from the Germano identity [19] that relates SGS stresses at different scales

$$L_{ij} = T_{ij} - \hat{\sigma}_{ij} = \hat{u}_i \hat{u}_j - \hat{u}_j \hat{u}_i,$$  \quad (5)

where $T_{ij}$ is the SGS stress at a scale $2\Delta$ and $L_{ij}$ is the resolved SGS stress tensor defined from scales intermediate between $\Delta$ and $2\Delta$. Filtering at a scale $2\Delta$ is denoted by a hat ('). $L_{ij}$ can be computed exactly from the resolved velocity field. Using the Smagorinsky model to express the deviatoric SGS stresses $T_{ij}$ and $L_{ij}$ yields an error induced by the use of the Smagorinsky model.

$$\epsilon_{ij} = L_{ij}^D - (T_{ij}^D - \hat{\tau}_{ij}) = L_{ij}^D - c_{\Delta}^2 M_{ij}, \quad (6)$$

where $M_{ij} = 2\Delta^2 \left[ S_{ij} - 4\beta \hat{\sigma} \hat{u}_i \hat{u}_j \right]$. In the Lagrangian SGS model, the coefficient $c_{\Delta}$ is obtained by minimizing the weighted time average of the squared error over pathlines. Using an exponential weighting function in time, as well as a simplified numerical treatment of the resulting relaxation transport equations (see [1] for details), we obtain

$$c_{\Delta}^2 = \frac{f_{LM}}{f_{MM}}, \quad (7)$$

where

$$f_{MM}^n(x) = \epsilon \left[ M_{ij} M_{ij} \right]^{n+1}(x) + (1-\epsilon) f_{MM}^n (x - \bar{u}^n \Delta t);$$

$$f_{LM}^n(x) = \epsilon \left[ L_{ij} M_{ij} \right]^{n+1}(x) + (1-\epsilon) f_{LM}^n (x - \bar{u}^n \Delta t);$$

$$\epsilon = \frac{\Delta t / T^n \cdot 1 + \Delta t / T^n}{T^n = 1.5 \Delta (f_{LM}^n f_{MM}^n)^{-1/8}} \text{ is the relaxation time scale used in the exponential weighting (allowing the model to allocate a higher weight to its recent history), and } H(x) \text{ is the ramp function.}$$

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$$f_{LM}^n(x) = \epsilon \left[ L_{ij} M_{ij} \right]^{n+1}(x) + (1-\epsilon) f_{LM}^n (x - \bar{u}^n \Delta t);$$

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function needed to ensure that small numerical errors do not lead to negative $f_{LM}$.

**LES OVER HETEROGENEOUS SURFACES**

Several simulations are performed to understand the effect of surface heterogeneity on the flow. The characteristics of the surface are varied and, for each simulation, the blending height ($h_b$) and effective surface roughness ($z_{o,e}$) are determined from LES results. Fig. 1 depicts the simulation domain and the main parameters used in the simulations; it illustrates an example with two unequal patches.

A pseudo-spectral approach is used in the horizontal directions and a second order accurate centered-differences scheme in the vertical along with a staggered grid. Time advancement is performed with the fully explicit second order accurate Adams-Bashforth scheme. The dynamic computation of the coefficient at every time step is not necessary; the time-step for the Lagrangian dynamic model ($\Delta t$ in Eq.7) is chosen to be 5 times larger than the main LES timestep, greatly reducing the central processing unit (CPU) time requirements. Tests with a smaller time step $\Delta t$ are performed to assure that this does not affect the model results. The bottom boundary condition is imposed using a modified version of the local law-of-the-wall to ensure that the average of the imposed wall stress is not erroneously high due to the local formulation. This is done using

$$\tau_w(x,y) = - \left[ \frac{\kappa}{\ln(z/z_0)} \right] \left[ \left( \frac{\bar{u}(x,y,dz/2)}{u(x,y,dz/2)} \right)^2 + \left( \frac{\bar{v}(x,y,dz/2)}{v(x,y,dz/2)} \right)^2 \right], \quad (8)$$

where $u$ and $v$ are the horizontal velocity components, $\kappa$ is the von-Kraman constant, and $z_0$ is the surface roughness.

A total of 28 simulations are performed; the simulations fall under 2 groups. All simulation parameters remain unchanged except the scale and roughness of the surface patches. The values of roughness length $z_o$ are varied from $10^{-6}H$ to $10^{-3}H$. In each case, only two roughness values $z_{o,1}$ and $z_{o,2}$ are used. The roughness ratio $z_{o,2}/z_{o,1}$ is set to 10 or 100. The patch length $L_p$ varies between 0.21H and 3.14H; in addition, a homogeneous surface simulation with an effectively infinite patch length (due to the periodic boundary conditions in the horizontal directions) is performed.

The first group (A,B,C,D) consists of simulations with varying number of patches of equal size; therefore, the patch length varies between simulations but is fixed within a single simulation. For more information about the details and result from this group refer to Bou-Zeid et al. [4]. The second group (SR) consists of simulations where the patch length varies within a single simulation; i.e. surfaces with unequal-sized patches. Table 1 details the simulations parameters.

<table>
<thead>
<tr>
<th>Simulation</th>
<th># of patches</th>
<th>$L_{p,1}/H$</th>
<th>$L_{p,2}/H$</th>
<th>$z_{o,1}/H$, $z_{o,2}/H$</th>
<th>$z_{o,e,LES}/H$</th>
<th>$h_b,LES/H$</th>
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<tr>
<td>A1</td>
<td>1</td>
<td>Infinite</td>
<td>$10^{-6}$</td>
<td>-</td>
<td>0.101</td>
<td>-</td>
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<tr>
<td>A2</td>
<td>2</td>
<td>3.14</td>
<td>3.14</td>
<td>$2.5 \times 10^{-5}$, $2.5 \times 10^{-4}$</td>
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<td>0.285</td>
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<tr>
<td>A4</td>
<td>4</td>
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<td>1.57</td>
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<td>0.125</td>
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<tr>
<td>A8</td>
<td>8</td>
<td>0.785</td>
<td>0.785</td>
<td>$2.5 \times 10^{-5}$, $2.5 \times 10^{-4}$</td>
<td>0.108</td>
<td>0.100</td>
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<tr>
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<td>12</td>
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<td>0.523</td>
<td>$2.5 \times 10^{-5}$, $2.5 \times 10^{-4}$</td>
<td>0.115</td>
<td>0.075</td>
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<tr>
<td>A20</td>
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<td>0.314</td>
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<td>0.209</td>
<td>$2.5 \times 10^{-5}$, $2.5 \times 10^{-4}$</td>
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<td>0.040</td>
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<tr>
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<td>3.14</td>
<td>$10^{-4}$, $10^{-3}$</td>
<td>0.366</td>
<td>0.315</td>
</tr>
<tr>
<td>B4</td>
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<td>1.57</td>
<td>$10^{-4}$, $10^{-3}$</td>
<td>0.372</td>
<td>0.190</td>
</tr>
<tr>
<td>B8</td>
<td>8</td>
<td>0.785</td>
<td>0.785</td>
<td>$10^{-4}$, $10^{-3}$</td>
<td>0.402</td>
<td>0.120</td>
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<tr>
<td>B20</td>
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<td>0.314</td>
<td>$10^{-4}$, $10^{-3}$</td>
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<td>3.14</td>
<td>$10^{-5}$, $10^{-3}$</td>
<td>0.229</td>
<td>0.290</td>
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<tr>
<td>C4</td>
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<td>1.57</td>
<td>$10^{-5}$, $10^{-3}$</td>
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<td>0.180</td>
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<tr>
<td>C8</td>
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<td>0.785</td>
<td>$10^{-5}$, $10^{-3}$</td>
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<tr>
<td>C20</td>
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<td>0.314</td>
<td>$10^{-5}$, $10^{-3}$</td>
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<td>0.070</td>
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<tr>
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<td>3.14</td>
<td>$10^{-6}$, $10^{-4}$</td>
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<td>0.250</td>
</tr>
<tr>
<td>D4</td>
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<td>1.57</td>
<td>$10^{-6}$, $10^{-4}$</td>
<td>0.0256</td>
<td>0.150</td>
</tr>
<tr>
<td>D8</td>
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<td>0.785</td>
<td>$10^{-6}$, $10^{-4}$</td>
<td>0.0300</td>
<td>0.088</td>
</tr>
<tr>
<td>D20</td>
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<td>0.314</td>
<td>0.314</td>
<td>$10^{-6}$, $10^{-4}$</td>
<td>0.0339</td>
<td>0.050</td>
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<td>$10^{-3}$, $10^{-4}$</td>
<td>0.090</td>
<td>0.225</td>
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<td>5.525</td>
<td>$10^{-5}$, $10^{-4}$</td>
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<td>0.230</td>
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<td>4.71</td>
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<td>0.44</td>
<td>$10^{-5}$, $10^{-4}$</td>
<td>0.0137</td>
<td>0.310</td>
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</table>

For the first group, finding a length scale $L_p$ that characterizes the size of the patches is relatively straightforward since all patches in a single simulation are of equal size, this patch length is used as the characteristic heterogeneity or patch scale $L_p$. For the second group, the average of the streamwise lengths of the patches is used for $L_p$. This method is simple in the present case but might be
problematic to use in simulations where the streamwise direction varies.

Results from Simulation A1 were used to validate the code with the SGS model (previous versions of the code were validated with other SGS model, see Albertson and Parlange [15, 16] and Porté-Agel et al. [21]). One of the main functions of an SGS model being to dissipate the right amount of energy, it is crucial that an SGS model produces correct energy spectra for a given flow. In turbulent flow over rough walls, scaling arguments and experimental results suggest that the streamwise spectra should display a slope of -1 in the production range. In the inertial subrange, the Kolmogorov spectrum should be reproduced with its characteristic -5/3 slope. Fig. 2 depicts the streamwise spectra for simulation A1; the expected slopes are well reproduced by the Lagrangian dynamic scale-dependent SGS model.

Figure 2. $u$ velocity spectra versus $k_1z$; $k_1$ is the stream-wise wave number and $z$ is the height. Each line represents the data for a selected height (at $z/H$ of 0.0042, 0.0126, 0.021, 0.046, 0.130, or 0.267). The numerical cost of the Lagrangian dynamic model exceeds the cost of the plan averaged scale-dependent dynamic model by about 6 percent only, mainly because the $c_s$ is recomputed only every fifth time step. To justify the use of this more complicated but local $c_s$ determination, the Smagorinsky coefficient has to be sensitive to the heterogeneity of the surface. To test this, the value of $c_s$ averaged in the cross-stream direction is plotted versus $x$ and $z$ in Fig. 3 for simulation A2 (2 patches). The coefficient is divided by its average over horizontal planes to remove the effect of vertical variations and detect the sensitivity to horizontal variations. As is clear in Fig. 3, the value of the coefficient varies by up to 50 percent between the high-roughness and low-roughness patches near the ground and the effect of surface heterogeneity extends up to 0.05H into the domain. This illustrates the benefits of performing a local determination of the Smagorinsky coefficient in LES of complex flows.

Figure 3. Sensitivity of the $c_s$ coefficient to surface roughness (simulation A2)

The effective surface roughness ($z_{o,e}$) is determined by a least square fit between the measure streamwise velocity and the law of the wall expression (Fig. 4); the first six grid points from the wall are used in the fitting. For more details about the methods used to determine these parameters refer to Bou-Zeid et al. [4]. Results are presented in Table 1.

Figure 4. Equivalent surface roughness determined from velocity profiles, the straight line are the least square fits for the first six points

The blending height ($h_b$) is determined by computing the upper and lower quartiles of the mean profiles of the streamwise velocity at each height averaged over time and cross-stream direction $y$. Then, the vertical profiles of these quartiles are plotted and the blending height is defined as the height where the quartiles display a neck (Fig. 5). The determined values of $h_b$ are shown in Table 1. The blending heights thus determined are plotted versus the length of the high roughness patch; results are shown in Fig. 6. The data was also plotted versus the low-roughness patch scale and the heterogeneity characteristic scale. The data was scattered in all plots.
INTERNAL BOUNDARY LAYER GROWTH

As the flow passes from one patch to another, an internal boundary layer (IBLs) emanates from the boundary of the patch. The flow inside the IBL is controlled by the underlying surface while the flow outside is controlled by the upstream patch and the flow in the outer region. The blending of surface heterogeneities by turbulent mixing in the flow is closely related to the formation and development of the internal boundary layers. As the IBL develops downstream and into the outer flow, its outermost parts are mixed by the turbulent eddies. The signature of the surface weakens as the distance from the wall increases and a point is reached where the IBL top is completely mixed with the outer flow and other IBLs, above that point the flow is statistically homogeneous in the wall-parallel direction.

To better understand the physical mechanisms controlling the growth of the IBL, a classic scaling argument was revisited [4]. Briefly reviewing, this approach estimates the growth of the boundary layer using a diffusion analogy. Scaling arguments yield:

\[
\frac{\partial \delta_{\text{IBL}}}{\partial x} - \text{upward diffusion velocity} \sim \frac{\langle u \rangle}{\langle \delta_{\text{IBL}} \rangle}\text{streamwise convection velocity} \sim \frac{w_{\text{rms}}}{\langle u \rangle} \quad (9)
\]

where \(x\) is the downstream distance, \(\langle u \rangle\) the mean stream-wise velocity at the top of the IBL, and \(w_{\text{rms}}\) the standard deviation (the root mean square) of the vertical velocity. \(w_{\text{rms}}\) can be approximated by \(u^*\) (from LES data, for the lower 20% of the ABL, \(w_{\text{rms}} \sim 0.95u^*\)) whereas the mean stream-wise velocity can be estimated from an effective log profile. Developing this scaling equation and integrating from \((\delta_{\text{IBL}} = z_{o,e} \text{ at } x = 0)\) to \((\delta_{\text{IBL}} \text{ at } x)\) yields the following relation between the IBL depth and the effective surface roughness \(z_{o,e}\):

\[
\ln \left( \frac{\delta_{\text{IBL}}}{z_{o,e}} \right) - 1 = C \kappa x \quad (10)
\]

\(C\) is an empirical constant, \(\kappa\) the von-Karman constant. \(\delta_{\text{IBL}}\) determined from this equation can be compared to \(\delta_{\text{IBL}}\) determined from LES (using the \(\frac{d\langle u \rangle}{dz} = \frac{d\langle u \rangle}{dx} / \frac{dz}{dx}\) method) to determined the empirical coefficient \(C\). As shown in Bou-Zeid et al. [4], a value of 0.85 for \(C\) can successfully predict \(\delta_{\text{IBL}}\).

BLENDING HEIGHT

Extension of the IBL growth equation to predict blending height can be successful if the downstream distance at which the IBL top reaches the blending height can be determined. To explore this, contour plots of the non-dimensional \(u\) gradient \(\kappa^2u^*/(d\langle u \rangle/dz)\) were inspected along with \(\delta_{\text{IBL}}\) determined from Eq. (10). In all simulations, when the IBL is allowed to grow until a downstream distance roughly equal to twice the patch length, the blending height is reached (Bou-Zeid et al., 2004). Hence, the blending height can be defined through

\[
h_b = \delta_{\text{IBL}}(x = \alpha L_p) \text{ with } \alpha = 2 \quad (11)
\]

\(\delta_{\text{IBL}}\) determined from this equation can be compared to \(\delta_{\text{IBL}}\) determined from LES (using the \(\frac{d\langle u \rangle}{dz} = \frac{d\langle u \rangle}{dx} / \frac{dz}{dx}\) method) to determined the empirical coefficient \(C\). As shown in Bou-Zeid et al. [4], a value of 0.85 for \(C\) can successfully predict \(\delta_{\text{IBL}}\).

\[
h_b \left[ \ln \left( \frac{h_b}{z_{o,e}} \right) - 1 \right] = 0.85 \kappa (2L_p) \quad (12)
\]

Fig. 7 compares the \(h_b\) predicted by this equation to the values determined from LES data; it depicts blending height
versus patch length (divided by the equivalent surface roughness to reduce the number of variables to 2, \( h_b/z_{o,e} \) and \( L_p/z_{o,e} \)). The symbols are the heights determined from LES data; the solid line is the prediction of Eq. (12). The comparison is quite good indicating that Eq. (12) can be used to predict the blending height for all patch cases considered in this study.

![Figure 7. Normalized mixing height versus patch length for all simulations fitted with a curve of the form \( h_b[\ln(h_b/z_o)-1]=1.7 \kappa L_p \).](image)

EFFECTIVE SURFACE ROUGHNESS

Eq. (12) used to compute the blending height involves the value of the equivalent surface roughness \( z_{o,e} \). The values for the surface roughness used till now were obtained from LES data; however, a parameterization for \( z_{o,e} \) is required to allow computation of the blending height without any need for flow measurements; moreover, a parameterization of \( z_{o,e} \) is important by itself for the reasons discussed in the introduction.

The proposed approach is based on work by Wieringa [25], Mason [26], and Claussen [3]. A slight modification, that becomes important for small patch scales, was proposed and tested versus LES results by Bou-Zeid et al. [4]; that study focused on the first group of simulations A-D. In this paper, tests are performed with simulations where the patch scales can vary in a single simulation; thus getting closer to realistic situations. Future work will consider patches with random sizes and roughness heights.

The net friction force over a heterogeneous area is written as the sum of friction forces over each patch in the area

\[
A_{\text{total}} \tau_{\text{total}} = \sum_{i=1}^{N} A_i \tau_i, \tag{13}
\]

where \( N \) is the number of patches, \( A_{\text{total}} \) is the total area considered, and \( A_i \) is the surface of patch \( i \). If the patches are assumed to be big enough so that the flow is in equilibrium with the underlying surface over most of the patch, the average surface stress over a patch can be expressed using the law-of-the-wall using

\[
\left( \frac{u_e \kappa}{\ln \left( \frac{z}{z_{o,e}} \right)} \right)^2 = \sum_{i=1}^{N} f_i \left( \frac{u_i \kappa}{\ln \left( \frac{z}{z_{o,i}} \right)} \right)^2, \tag{14}
\]

where \( u_e \) is the velocity averaged over the whole domain and \( z_{o,e} \) is the equivalent surface roughness that is to be determined by this procedure. Moreover, \( u_i \) and \( z_{o,i} \) are the equilibrium velocity and the surface roughness of patch \( i \) and \( f_i \) is the fraction of the total area covered by patch \( i \) (\( f_i = A_i/A_{\text{total}} \)). Following references [25] and [26], the above equation is evaluated at the blending height \( h_b \) where one may assume that the flow velocity is homogeneous over all the area (\( u_i \approx u_e \) for any patch). The equation reduces to

\[
\left( \frac{1}{\ln \left( \frac{h_b}{z_{o,e}} \right)} \right)^2 = \sum_{i=1}^{N} f_i \left( \frac{1}{\ln \left( \frac{h_b}{z_{o,i}} \right)} \right)^2. \tag{15}
\]

Eqs. (12) and (15) can be combined to yield an equation for \( h_b \) that has to be solved iteratively:

\[
\left( \frac{h_b}{1.7 \kappa L_p + h_b} \right)^2 = \sum_{i=1}^{N} f_i \left( \frac{1}{\ln \left( \frac{h_b}{z_{o,i}} \right)} \right)^2. \tag{16}
\]
The value of $z_{o,e}$ can then be computed from Eq. (12) according to:

$$z_{o,e} = h_b \exp \left[ \frac{-1.7}{h_b} \frac{L_p}{L_h} - 1 \right].$$

(17)

Next, this model (i.e. Eqs. (16) and (17)) is used to estimate the equivalent surface roughness for the LES runs, and the results are compared to the effective surface roughness determined from LES data. Fig. 8 presents the ratio $z_{o,e}^{\text{model}}/z_{o,e}^{\text{LES}}$ when plotted versus $z_{o,e}^{\text{LES}}$. A value of 1 (thick black line) indicates a perfect model. All model estimates are within 25 percent of the LES results suggesting that the proposed parameterization is quite successful in predicting the equivalent surface roughness for simple patch distributions. In future work, tests will be performed to see if the performance of the parameterization is as satisfactory with highly random distributions of roughness. The more realistic configurations that will be tested include patches of different sizes and complex configurations. An important challenge in such situation is to be able to find a single characteristic scale for surface heterogeneity. See reference [27] for an interesting discussion of approaches that can be used to obtain such a characteristic patch length.

**CONCLUSIONS**

LES was used to simulate turbulent wall-bounded flows with heterogeneous roughness distributions. Two groups of simulation were performed with varying roughness heights and heterogeneity scales. Plots of the dynamically-determined Smagorinsky coefficient used in the SGS model indicate that it is quite sensitive to the surface roughness thus validating the need for a model where $c_s$ is determined locally such as the Lagrangian model implemented here. A parameterization for internal boundary layer growth rate was developed and used to predict the blending height, and combined with an equation for surface stresses. This resulted in a system of 2 equations for the blending height and the effective surface roughness. The predictions of this parameterization were successfully compared to LES values suggesting that the parameterization can be used to predict effective surface roughness from basic patch characteristics, namely patch scale and roughness heights. The effective surface roughness heights from the parameterization were within 25 percent of LES values.

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