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<td><a href="mailto:corey-markfort@uiowa.edu">corey-markfort@uiowa.edu</a></td>
</tr>
<tr>
<td>Raúl Bayoán Cal</td>
<td><a href="mailto:rcal@pdx.edu">rcal@pdx.edu</a></td>
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Increasing VAWT Wind Farm Power Density using Synergistic Clustering

Seyed Hossein Hezaveh1, Elie Bou-Zeid1,*, John Dabiri2, Matthias Kinzel3, Gerard Cortina4, Luigi Martinelli5

1 Department of Civil and Environmental Engineering, Princeton University
2 Department of Civil and Environmental Engineering and Department of Mechanical Engineering, Stanford University
3 Graduate Aerospace Laboratories, California Institute of Technology
4 Department of Mechanical Engineering, the University of Utah
5 Department of Mechanical and Aerospace Engineering, Princeton University

*Corresponding author: ebouzeid@princeton.edu

Abstract

Vertical axis wind turbines (VAWTs) are being reconsidered as a complementary technology to the more widely horizontal axis wind turbines (HAWTs) due to their unique suitability for offshore deployments. In addition, field experiments have confirmed that VAWTs can interact synergistically to enhance and increase the total power production when placed in close proximity. In this paper, we use an actuator line model in a large eddy simulation code (ALM-LES) to test novel VAWT farm configuration that exploits these synergistic interactions. We first design clusters with three turbines each that exploit the omni-directional capability of VAWTs and optimize the distance between the clustered turbines. We then configure farms based on clusters, rather than individual VAWTs. The simulations confirm that VAWTs have a positive influence on each other when packed in well-designed clusters: such configurations can increase the power generation of a single turbine by about 10 percent. In addition, the cluster designs allow for closer turbine spacing resulting in about three times the number of turbines for a given land area compared to conventional configurations. Therefore, both the turbine and wind farm efficiencies are improved, leading to an increase in wind-farm averaged power density by as much as 60 percent.
Keywords: Large Eddy Simulation, Vertical Axis Wind Turbines, Wind Energy, Wind Farms, Wind Turbine Wakes

1 Introduction

Despite the concerted effort to improve energy efficiency and decouple economic growth from energy consumption, the U.S. Energy Information Administration projects that global total energy consumption will grow by about 45% between 2015 and 2040 (U.S. Energy Information Agency, 2013). Mitigating the concomitant large increase in greenhouse gasses (GHG) emissions necessitates exploring alternative low-GHG energy sources particularly that the majority of the current fossil-based energy resources are finite and have other adverse side effects on the environment. Wind energy is expected to be one of the primary sources of clean, renewable energy that would allow a fast transition away from fossil-based energy. In the US for example, wind is projected to contribute around 20% of electric energy by the year 2030 (Marquis et al., 2011). As a result, larger and larger wind farms are being deployed; however, the continued spread and expansion of these farms pose a challenge since the required land area will increase. A major goal of current research is thus to increase the wind farm power density, i.e. how much energy is produced per unit land area used.

In a wind farm, turbines should be far enough apart to allow wind speeds to recover after deceleration by the upwind generator (by lateral or vertical momentum entrainment (Cortina et al., 2016), and to reduce the fatigue load generated by turbulence from the upstream turbines and increase the turbine lifetime (Chamorro & Porté-Agel, 2009). The large majority of existing farms are using horizontal axis wind turbines, HAWTs. The behaviour of HAWTs in large wind farms, and the required spacing between them have been extensively studied (Troldborg &
Sørensen, 2014; Wu & Porté-Agel, 2012). Calaf et al. (Calaf et al., 2010, 2011) investigated the fully developed horizontal axis wind turbine array boundary layer (WTABL) and quantified the vertical transport of momentum and kinetic energy across the boundary layer. They have shown that in large wind farms the kinetic energy regeneration is mainly from downward vertical fluxes across the plane delineating the top of the farm, unlike farms with a limited number of wind turbine rows where the streamwise advection of kinetic energy dominates. The concept of WTABL is valid for wind farms whose length is an order of magnitude larger than the height of atmospheric boundary layer (ABL) since the influence of such farms would extend all the way to the top of the ABL. Meyers and Meneveau (2010) used an actuator disk model and large eddy simulations (LES) to model large HAWT wind farms and understand their interaction with the ABL. They have shown that a staggered wind farm can extract 5% more power than an aligned configuration. In a follow-up study in 2012 (Meyers & Meneveau, 2012), the same authors investigated the optimization of turbine spacing in fully developed wind farms; they showed that varying the ratio of land cost to turbine cost in the financial optimization analysis (minimizing power per unit cost) results in different optimal spacing. Meyers and Meneveau’s results indicate that the optimal turbine spacing is higher than what is currently being used in HAWT wind farms. Recently, it has also been shown that the highest mean wind farm power is highly dependent on the alignment angle (of turbine rows relative to the wind) and, contrary to previous results, the optimal angle is significantly smaller than the one in a perfectly staggered farm (Stevens et al., 2014). In wind farm sites with a dominant wind direction, these findings can be implemented to improve wind farm performance.

These papers and the large majority of previous research and development efforts focused on wind farms consisting of HAWTs (Chamorro & Porté-Agel, 2010; Lu & Porté-Agel, 2011;
Meyers & Meneveau, 2012; Yu-ting, 2011). However, recent investigations by Dabiri (2011) have suggested the possibility of an order of magnitude increase in power densities for wind farms when vertical axis wind turbines (VAWTs) are used. VAWTs have vertical blades that rotate about a vertical axis, and hence VAWT wakes (Bachant et al., 2016) and the flow in a VAWT farm are distinctively different from their HAWT counterparts. This increase in power density can be achieved by configuring VAWT farms with a closer spacing to better exploit the flow patterns created by upstream turbines. In their studies, they performed experiments on various counter-rotating configurations of 9-m tall VAWTs and demonstrated that, unlike the typical performance reduction of HAWTs with small spacing, there is an increase in VAWTs performance when adjacent turbines are arranged to interact synergistically. However, high experimental costs and time requirements prevent the extension of these field investigations to large farm scales or the assessment of a large number of configurations. The previous findings thus only pertain to a limited number of turbines where the mean kinetic energy (MKE) is primarily replenished by streamwise advection and cross-stream turbulent transport, rather than by vertical transport as in large farms. The aim of this paper is, therefore, to bridge this research gap and assess the feasibility of increasing power density in large VAWT farm using synergistic clustering of turbines. Following the recent results provided in Hezaveh et al. (2016), where a large eddy simulation code for VAWTs was extensively validated and the flow recovery in the wake of a single turbine investigated, we here simulate the interactions of multiple VAWTs in small clusters, and subsequently use these clusters to design large VAWT farms.

2 Numerical Model

In order to investigate VAWTs in the ABL, the LES code with an actuator line model (ALM-
LES) presented and validated in Hezaveh et al. (2016) was used. In this code, which has also been widely validated for flows around HAWTs (Chamorro & Porté-Agel, 2009; Marquis et al., 2011) and other flows (Chamorro & Porté-Agel, 2010; Dabiri, 2011; Hezaveh et al., 2016), the continuity and Navier-Stokes equations are solved at each time step for the large, resolved scales assuming an incompressible flow with a mean in vertical hydrostatic equilibrium (Bou-Zeid et al., 2005):

\[ \frac{\partial \tilde{u}_i}{\partial x_i} = 0 \],

(1)

\[ \frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \left( \frac{\partial \tilde{u}_i}{\partial x_j} - \frac{\partial \tilde{u}_j}{\partial x_i} \right) = -\frac{1}{\rho} \frac{\partial \tilde{p}^*}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j} + F_{i} + F_{i}' \].

(2)

In the equations above, \( \tilde{u}_i \) is the resolved velocity vector with the tilde denoting a filtered quantity; \((u, v, w)\) are its streamwise, spanwise and vertical components, respectively. This instantaneous velocity vector will be decomposed into a mean \( U_i \) and a resolved perturbation \( u'_i \).

\( x_i \) is the position vector with components \((x, y, z)\) in the streamwise, spanwise and vertical directions respectively; \( \tilde{p}^* \) is a modified pressure that includes the resolved and subgrid scale turbulent kinetic energies; \( \rho \) is the air density; \( F_i \) is the mean pressure gradient driving the flow; \( \tau_{ij} \) is the deviatoric subgrid scale stress tensor; and \( F_{i}' \) represents the aerodynamic forces of the turbine blades on the air flow. At each time step \( F_{i}' \) is computed using the ALM as detailed in Hezaveh et al. (2016). The horizontal boundary conditions are numerically periodic, but non-periodic flows can be simulated using an inlet sponge region as detailed later. At the top boundary, zero vertical velocity and zero shear stress are imposed. The bottom boundary has
zero vertical velocity, while the surface shear stress is imposed using an equilibrium log-law wall
model with a wall roughness of $z_0 = 10^{-6} z_i$, where $z_i$ is the depth of computational domain used
to normalize all length scales in the code ($z_i = 25m$). The details of the wall and subgrid scale
model are provided in Bou-Zeid et al., 2005. The model details are summarized for the reader in
Fig. 1: an angle of attack ($\alpha$) is first computed by knowing the location of each blade represented
as vertical line in the ALM, the incoming velocity ($U_{\infty}$), and the rotational speed of the turbine
($\omega$). This then allows us to obtain the lift and drag force coefficients, $C_L$ and $C_D$ respectively
(from experimental data, blade-resolving Reynolds averaged simulations, or tabulated airfoil data
after applying a dynamic stall correction, as detailed in Hezaveh et al., 2016). $C_L$ and $C_D$ are then
used to compute the normal and tangential force coefficients, $C_N$ and $C_T$ respectively:

\begin{align}
C_N &= |C_L| \cos \alpha + |C_D| \sin \alpha , \quad (3) \\
C_T &= |C_L| \sin \alpha - |C_D| \cos \alpha , \quad (4)
\end{align}

which are then used to compute the corresponding forces

\begin{align}
dF_N(\theta) &= \frac{1}{2} \rho c V_{rel}^2 C_N dz , \quad (5) \\
dF_T(\theta) &= \frac{1}{2} \rho c V_{rel}^2 C_T dz , \quad (6)
\end{align}

where $c$ is the blade chord length, $\theta$ the azimuthal angle at a given time, $\rho$ is air density and $dz$ is
the vertical length of blade occupying each LES grid cell. Finally, the forces in the Cartesian
coordinate system of the LES are obtained as:
In order to find $C_L$ and $C_D$ in this paper, actual dynamic curves for lift and drag forces determined experimentally were implemented into the numerical code. Claessens (2006) has conducted several numerical and wind tunnel experiments at various Reynolds numbers and tabulated the lift and drag curves for different airfoil blade types. Fig. 2 depicts the measured values that included the dynamic stall effect (accounted for in the wind tunnel measurements).

Based on the sign of the instantaneous rate of change of the angle of attack, different paths emerge in these curves in Fig. 2; $C_{Lm+}$ versus $C_{Lm-}$ and $C_{Dm+}$ versus $C_{Dm-}$. These results are for the DU-06-W-200 blade type, which is redesigned from NACA 0018 airfoil and is 2% thicker, and cambered rather than symmetric (Claessens, 2006). This blade type and the experimental results reported in Fig. 2 are used throughout this study.

\[ dF_x = (dF_N(\theta) \cos \theta + dF_T(\theta) \sin \theta) \]  
\[ dF_y = (dF_N(\theta) \sin \theta - dF_T(\theta) \cos \theta) \]
Fig. 2 Dynamic $C_L$ and $C_D$ for the DU 06-W-200 blade type as measured by Claessens, 2006. The + subscript is for $\dot{\alpha} > 0$ and the – is for $\dot{\alpha} < 0$

3 Results and Discussions

3.1 Validation against field measurements of counter-rotating VAWTs

As mentioned before, Dabiri (2011) performed several field measurements with different configurations of 9-m tall VAWTs using the same blade type and rotor configuration we adopt here. They have shown that, by using counter rotating VAWTs and special configurations, the VAWTs can exploit the flow deflection from upwind adjacent rotors and there is a potential of an order of magnitude increase in power density. To complement the previous validation of this ALM-LES code performed against laboratory experiments (Hezaveh et al., 2016), and to ensure that the simulations accurately represent the flow in between multiple turbines and therefore within and in the wake of turbine clusters, we will compare our LES results to the field measured data described in Dabiri (2011) for two adjacent counter-rotating turbines. This is the first validation of our model against data from real-sized VAWT field measurements, and in fact, to
the best of our knowledge, the first validation of any VAWT ALM-LES against field data.

The experimental setup details are presented in Table 1, and the schematic configuration is shown in Fig. 3. The turbines are a modified version of a commercially available model (Dabiri, 2011) and they were placed $1.6D$ ($D$ is the rotor diameter) apart. The velocity profiles were measured at 16 points with streamwise coordinates (relative to the line joining the centre of the turbines) $x = -15D, -1.5D, 2D, 8D$ and elevations above ground of $z = 3m, 5m, 7m$ and $9m$. All of the velocities are normalized using a 10 m meteorological tower in the vicinity of the experiments (Araya et al., 2014). The computation domain was $L_x \times L_y \times L_z = 31.2m \times 15.6m \times 25m$, respectively spanned by $128 \times 64 \times 192$ grid nodes.

In order to match the inflow conditions such as turbulence intensity and mean upstream velocity profile in LES to the observed field data, a precursor periodic simulation was run to generate the inflow. The surface roughness and friction velocity of this precursor simulation were calibrated to yield the experimentally-observed log-law velocity profile. Then, $y-z$ slices of instantaneous velocity and pressure were saved at each time step and fed to the simulation with the turbines as upwind inflow.

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<th>Variable</th>
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<th>Value</th>
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<tr>
<td>Number of blades per turbine</td>
<td>$N$</td>
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<tr>
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<td>$D$ [m]</td>
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<td>Blade chord length</td>
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</tr>
<tr>
<td>Airfoil section type</td>
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<tr>
<td>Tip speed ratio (selected at maximum $C_P$)</td>
<td>$\lambda$</td>
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</tr>
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Table 1 Turbine characteristics from Dabiri, 2011
The results are shown in Fig. 4, and it is clear that the ALM-LES is capable of closely reproducing the wake generated by the interactions of the two counter rotating VAWTs (the blades move towards the back when facing the other turbine, such that the flow acceleration in between the two rotors is maximized). We should highlight that it was essential to provide the LES with the correct inflow (left panel of Fig. 4, from the precursor simulation) for the experimental profiles near and behind the turbines (right 3 panels of Fig. 4) to be reproduced accurately. These results confirm that the ALM-LES model produces realistic wakes even where
turbines are interacting, and hence investigating large wind farms and VAWT clusters can proceed with confidence. It should also be mentioned that ALM-LES is capable of realistically capturing wake meandering, but this meandering will not appear in the figures presented in this paper since we only show mean velocities. Furthermore, the omission of the Coriolis force will not influence the result given the high Rossby number in the atmospheric surface layer. While Coriolis will induce Ekman turning, for the omni-directional VAWTs the effect on performance is smaller than for HAWTs.

![ALM-LES incoming velocity and wakes versus field measurements data](image)

**Fig. 4** ALM-LES incoming velocity and wakes versus field measurements data

### 3.2 VAWT Cluster Design

Clustering VAWTs in small arrangements was shown to have several advantageous implications for power generation (Dabiri, 2011). The global performance of the turbines is enhanced since the downstream turbines can benefit from the flow deflection effect and the resulting higher
velocity induced by upstream turbines. However, depending on the wind direction, compact clustering might also have negative effects when one turbine is mainly in the wake/shadow of an upstream one. For example, if two turbines are clustered together, the range of wind directions for which one of the turbines is affected by being in the shadow (partially or fully) of the other is

\[ 2\beta, \text{ where } \beta = \tan^{-1}(2D/L) \] (Fig. 5, left), \( L \) being the turbine spacing (centre to centre) in a cluster. On the other hand, when the wind is approximately perpendicular to the centre-to-centre axis, the higher induced velocity in between the two turbines is not being exploited in such cases.

![Diagram](image)

**Fig. 5** Wind directions in which VAWTs are in the wake of an upstream rotor for 2, 3 and 4 turbines (\( \gamma \approx \beta \))

By only introducing a third turbine, the number of wind directions where 2 turbines can directly shadow each other is increased to \( 6\beta \) (Fig. 5, middle). However, the third turbine can benefit from the higher velocity induced in between the two upstream ones or the two downstream rotors can benefit from the transverse flow deflection of the upstream turbine (depending on wind direction). This has the potential to result in a power production of these 3 turbines that is greater than the power from three distant non-interacting ones (this improvement will also depend on \( L/D \) as we will show). By increasing the number of turbines in the cluster beyond 3, the flow-
related benefits will drop and the number of wind directions where the turbines shadow each other will increase to \( n (n-1) \beta \), where \( n \) is the number of turbines in the cluster (e.g., Fig. 5, right). In Fig. 6, the variation of \( \beta \) with \( L/D \) and \( n \) for various clusters is shown. By increasing \( L/D \) of a cluster, the \( \beta \) value is reduced. On the other hand, increasing \( n \) results in higher \( \beta \). For \( n > 3 \), the \( \beta/2\pi \) value can become larger than 1, which indicates that there is no wind direction where the turbines are not casting shadows on each other. However, one can note that for \( L/D > 5 \), the differences between \( n = 2 \) and \( n = 3 \) are minor. Moreover, a clustering with higher \( n \) has the important benefit of using a smaller land area. Therefore, the most efficient design for a cluster when there is no dominant wind direction at the site seems to be a triangle \((n = 3)\) since it has a limited \( \beta \) while at the same time allowing for compact clustering and synergistic interaction between the turbines. Going to \( n = 4 \) almost doubles the shadowing angle \( \beta \), with no increase in the wind direction range for which synergistic interactions occur. Therefore, for the rest of this paper we will focus on triangular clusters.

In order to investigate the characteristics of the proposed triangular cluster design, we conduct a suite of large eddy simulations in a computational domain containing three of the same turbines defined in Table 1. The basic domain size is \( L_x \times L_y \times L_z = 72 \text{ m} \times 48\text{ m} \times 25\text{ m} = 60 \times 40 \times 20.8 \text{ D} \) and it is spanned by \( N_x \times N_y \times N_z = 288 \times 192 \times 192 \) grid nodes. These remain constant for the analyses in this subsection (except of the domain size sensitivity analysis detailed later). The power coefficient \( C_p \) of a single isolated turbine, as simulated by the LES, is 0.36. Since the wake deficit increases with \( D \) and decreases with the distance between the turbines \( L \). \( L/D \) is an important dimensionless number to consider. Furthermore it controls the shadow angle \( \beta \) as illustrated in Fig. 6. In order to investigate the optimal distance, various \( L/D \) ratios ranging from
2 to 8 were hence simulated.

However, before conducting these simulations the computational setup needed verification; therefore, for one fixed $L/D = 6$, an analysis of the sensitivity of the results to the domain size was performed (such that the domain size and number of grid points increase proportionally, and thus the grid resolution is unchanged). Two parameters were investigated for sensitivity to the domain size: the average cluster power coefficient $C_P$ and the wake velocity deficit values at 15D and 20D downstream of the cluster. The wake velocity deficit is averaged in time and over a $y$-$z$ rectangle that is aligned in the $x$ direction with the turbine cross-section projected area. As can be seen from Fig. 7, changes in domain length $L_x$ have little impact on the average $C_P$; changes in domain width $L_y$, on the other hand, can be significant when $L_y$ becomes very small (the crossflow area blocked by the turbines becomes large) and prevents correct sideways deflection.
of the streamlines. The figure suggests that a minimal $L_y \geq 40 D$ should be used, since increasing
the transverse domain size to $L_y = 54 D$ results in insignificant changes in the average $C_P$.
However, the velocity deficit values are also changing with $L_x$ (due to downstream boundary
condition effects). Based on the sensitivity analysis results in Fig. 7, a $L_x > 40D$ is required.
Therefore, a domain size of $L_x = 60 D$ by $L_y = 40 D$ is adopted for the single cluster simulations.
All these simulations were conducted using laminar logarithmic profile inflow. To assess the
influence of the turbulence levels in the inflow, a simulation was conducted using inflow planes
from a precursor periodic run. As can be seen in Fig. 7, using a turbulent inflow reduces the
deficit values at 15D and 20D downstream of clusters significantly, which can be explained by
the fact that the increase in turbulence intensity increases momentum entrainment into the wake
and speeds up its recovery.

![Diagram showing domain size sensitivity analysis. The adopted size is 60D×40D. $C_P = \text{Turbine Power} \left(1 - \frac{1}{2} \frac{AU_{\infty}^3}{\lambda} \right)$]
With the basic domain size set, simulations with triangle clusters with different $L/D$ ratios were conducted first using the unique wind direction of 60 degrees depicted in Fig. 7. Based on simulation results for different cases (see Fig. 8), it is obvious that by increasing $L/D$, the performance of the first/upwind turbine is increased due to the decrease in upstream blockage effect from turbines 2 and 3. Due to the rotation direction of the first turbine (shown in Fig. 7), which deviates the flow towards the third turbine, that third turbine has a slightly higher $C_P$ compared to the second turbine. On the other hand, Fig. 8 illustrates that the performances of second and third turbines first improve as $L/D$ increases from 2 to 3 then they plateau. However, when $L/D$ increases beyond 5, their $C_P$ decreases as they are less able to utilize the higher velocity induced from the flow deflection by the upstream rotor. The cluster-average $C_P$ (related to the upstream wind speed $U_\infty$) thus peaks at an intermediate $L/D$. The three cases with $L/D$ of 3, 4 and 5, which had the highest average $C_P$, were hence selected for further analysis.

\[ C_P = \text{rotor area} \]

\[ L/D \]

**Fig. 8** $C_P$ for each turbine in the cluster, and the averaged for the whole cluster, as a function of $L/D$
These analyses consisted of simulations where all parameters remain the same for a given \( L/D \), but with different incoming wind orientation. We aim to investigate omni-directionality of the proposed VAWT clusters, as well as to find the most efficient VAWT spacing averaged over all wind directions. Fig. 9 (a) shows the average \( C_p \) versus incoming wind direction. The case with \( L/D = 5 \) has the highest \( C_p \) averaged over all turbines for all wind directions. This is confirmed in Fig. 9 (b) that depicts influence of \( L/D \) on the \( C_p \) averaged over all wind directions and all turbines, and normalized by the \( C_p \) of a single isolated turbine. \( L/D = 5 \) performs better because the wind direction angles at which the VAWTs are casting shadows on downstream turbines (\( \beta \)) are reduced due increasing distance between turbines, as well as because wake recovery is improved when there is such shadowing. Finally, a key observation from Fig. 9 (b) is that the average \( C_p \) is about 10% higher than for a single isolated turbine when \( L/D = 5 \); this confirms that the synergistic interaction between closely-space turbines can indeed result in a higher overall power generation when exploited adequately.

**Fig. 9** a) Triangular cluster-average \( C_p \) versus wind direction. b) Cluster-average \( C_p \), also averaged over all wind directions, and normalized by \( C_p \) of a single isolated turbine (angled brackets denote averaging)
3.3 Cluster Wake Analysis

With the most efficient cluster design selected, we now turn our attention to the design of farms based on these clusters. An important parameter in designing and optimizing wind farms is the distance required for wind velocity and power recovery downstream of turbines (Hezaveh et al., 2016); this applies for layouts consisting of single turbines as well as of clusters (unless there is a dominant known wind direction). The wind velocity deficit \(1 - \frac{U(x, y, z)}{U_{\infty}(z)}\) was averaged over the y-z planes encompassing the whole cluster (projected wind-normal area) at varying x distances from the hub using data from the same runs described in the previous subsection. We also investigated various L/D to confirm that our choice of \(L/D = 5\) made based on the power output of an isolated cluster does not produce longer wakes than other L/D ratios. The result reported in Fig. 10 indicate that increasing the distance between the turbines in each triangle cluster significantly reduces the distance needed for the wind velocity to recover to 75% of upstream velocity \(U_{\infty}\). It is clear from the figure that the recovery distance is reduced from 25D for \(L/D = 3\) to 15D for \(L/D = 5\).

This is an important criterion for designing a wind farm, which further solidifies the selection of \(L/D = 5\). In a wind farm, it is important that downstream turbines are placed at distances were the available wind has recovered to at least 75% of its upstream undisturbed velocity so the power generation capacity of these turbines is not underutilized. Furthermore, as shown in the Fig. 10, by increasing the distance between the turbines in the clusters, the effect of incoming wind direction on the recovery distance is reduced. The \(L/D = 3\) recovery is sensitive to the change in incoming wind angle \(\zeta\); the recovery distance to 75% occurs anywhere between 18D and 28D as the wind angle changes. The recovery for \(L/D = 5\) on the other hand is much less sensitive to
wind direction and thus yields omnidirectional farms. The results also indicates that when designing farms based on clusters with $L/D = 5$, the required separation between clusters for $\approx 75\%$ recovery ranges between 15 and 18 $D$.

![Comparison between averaged velocity deficit at various $L/D$ and for different wind directions](image)

**Fig. 10** Comparison between averaged velocity deficit at various $L/D$ and for different wind directions

### 3.4 Cluster Wind Farm Design

Now we tackle the main question of the paper: can synergistic interactions between VAWTs increase wind farm power density. Practically, we need to investigate whether farms of
synergistic clusters perform better (produce more power per unit land or per unit invested cost) than two prototypical layout of wind farms, aligned and staggered regular arrays. Based on the size of the selected turbine and the results obtained in the previous section, 11 farm configurations were simulated. One configuration is illustrated in Fig. 11. The turbines are the same as the ones detailed in previous sections. The simulations in this section are all periodic (representing an infinite farm), with $N_x \times N_y \times N_z = 320 \times 160 \times 336$ nodes and $L_x \times L_y \times L_z = 96 \text{m} \times 48 \text{m} \times 32 \text{m}$.

Fig. 11 Schematic of the wind farm configuration with the VAWT triangular clusters, with turbine-to-turbine separation distances of $L = 5D$ and inter-cluster spacing of $20D$ for an aligned cluster configuration.
Six of the cases were prototypical wind farms with staggered or regular array configurations and with separation distances of 5D, 10D or 20D for each configuration; these configurations result in 128, 32 and 8 turbines in the computation domain, respectively. Four additional experiments were conducted with wind angles of 0 or 60 degrees, using staggered clusters with $L/D = 5$ within each cluster, and with inter-cluster distances of 20D or 10D for each wind angle, corresponding to 24 and 96 turbines in the domain, respectively. It should be mentioned that since the LES code uses periodic boundary conditions in both $x$ and $y$ directions, these simulations correspond to infinite wind farms with infinite number of turbines. Therefore, the number of turbines in the computational domain will not influence the results when normalized per turbine. Finally, one experiment is conducted using 20D spacing and aligned clusters with a 60 degree wind angle relative to the cluster. To visualize the differences in the flow patterns in these designs, the average streamwise velocities for a few selected configurations are shown in Fig. 12. The lower velocity in the staggered configurations reflect the higher power extraction resulting from a larger number of turbines, and therefore metrics that allow consistent comparison of these configurations are needed.

The most direct consistent metric is the average wind farm $C_P$, which is shown for different configurations in Fig. 13. The average wind farm $C_P$ is computed using as reference velocity the average streamwise wind speed in the whole wind farm volume containing the VAWTs (i.e. over a volume spanning a full $x$-$y$ plane and the $z$ domain from the bottom to the top of the blades). As anticipated the staggered cases have higher $C_P$ compared to the aligned ones, for both the clustered and regular designs. What is more interesting and relevant is that the clustered designs consistently produce higher power than the prototypical design for any spacing. As indicated previously, the $C_P$ for an isolated turbine is 0.36 and the cluster-staggered designs with an inter-
cluster spacing of 20D are surpassing this value over the whole wind farm for both wind angles. This is due to the gain in average $C_p$ that clusters allow, and the large inter-cluster spacing in this case that minimizes the penalty of being in the wake of the upstream cluster. By reducing the distance between clusters to 10D, the average $C_p$ drops, but remains significantly higher than for the corresponding regular wind farms.

![Streamwise velocity in wind farms with 10D horizontal spacings, mean wind from left to right](image)

**Fig. 12** Streamwise velocity in wind farms with 10D horizontal spacings, mean wind from left to right a) regular aligned, b) regular staggered, c) cluster staggered with 0 degree wind angle and d) cluster staggered with 60 degree wind angle

Another interesting aspect of the results is that the staggered configurations, even at small separation distances, consistently perform better than the aligned ones. The streamwise separation in the staggered 10D case for example is the same as the separation in the 20D aligned case, and yet the staggered 10D layout yields a higher $C_p$. One physical reason for this improved performance is that in the staggered cluster farms, in addition to the synergistic interactions within each cluster, the clusters themselves seem to also interact favourably. One can observe for
example in Fig. 12 (c) and (d) that two adjacent clusters accelerate the flow in between them, allowing the next staggered row to benefit from this flow deviation. This is exactly similar to the acceleration within a cluster but is now occurring in between clusters, suggesting a fractal attribute to these synergistic interactions (although with only two fractal generations here).

![Graph](image)

**Fig. 13** Average wind farm $C_p$ for various configurations

The results in Fig. 13, however, exclude an important difference between these periodic simulations. Due to higher drag forces exerted on the ABL in cases with higher densities (5D spacing) or cases with more efficient farm layouts, the required pressure gradient imposed in the simulations to yield a steady state mean flow will also be higher. In the LES, at each time step, the drag exerted on the ABL by the whole wind farm and the ground surface is computed and the
needed mean streamwise pressure gradient is imposed. This gradient reaches a steady state eventually, when the mean flow equilibrates. Due to this feature, the different cases have unique pressure gradients over the wind farm. This pressure gradient is an important parameter in such periodic simulations that are mimicking very large farms. In large farms, the wind speed (streamwise-averaged over the footprint of a row) is constant over most of the rows, and the main source of energy is this pressure drop rather than the upwind velocity (the latter being the main parameter for small farms). Thus, the kinetic energy that can be extracted in large farms scales with the pressure drop that scales with the total surface drag, while the advective velocity for the energy (to obtain power) is the constant wind speed. That implies that for large farms the total pressure drop along the farm (and thus the available energy) will be limited since these large farms influence pressure fields in the atmosphere significantly. Therefore, in order to be able to compare the various configurations without this pressure drop discrepancy, the following $C_P^*$, normalized per unit pressure gradient, is introduced:

$$
C_P^* = \frac{\text{Average Power per Turbine}}{\frac{1}{2} \rho A U_T^2}, \quad u_+^* = \sqrt{\frac{\Delta P_{(Loss)}}{L_z}},
$$

(9)

where $u_+^*$ is the root square of total mean surface drag (on ground + turbines), which is related to the total pressure drop $\Delta P_{(Loss)}$, $U_T$ is mean streamwise velocity over the wind farm volume (average of the domain containing the blades as defined before), and $A$ is the rotor area of a single turbine. The comparison of this new performance metric for the various configurations is presented in Fig. 14. Since $u_+^*$ is about 10 percent of horizontal velocity, $C_P^*$ is about 100 times $C_P$, and should not be interpreted as a classic power coefficient. Even after normalizing the total
power generated in these layouts by the exerted pressure gradient for each case, the cluster cases maintain the highest $C_p^*$. This means that these cases were able to extract more energy from the applied pressure gradient in the field compared to regular wind farms. The relative differences in the performance of the farms is expected to be closer to the differences depicted in Fig. 13 for smaller farms, and closer to the ones in Fig. 14 for larger farms.

A comparison of the power density per unit land area used for the various configurations was also performed. It confirmed that clustered designs increased the power density, validating the hypothesis of the paper. However, the results have the caveat that the power density will be invariably higher for very close spacing, even when the turbines in the farm are not being used efficiently (low $C_p$). Therefore, power density itself cannot be used a metric for optimizing...
farm layout. In order to have a more realistic and practical metric, the total capital cost per unit power generation is computed. Since the power generation for each farm is proportional to the sum of the $C_p$ values of all the individual turbines in a given lot area of fixed size, $C_{P,\text{total}}$, we use this sum for normalization instead of the actual power. The capital costs consist mainly of the cost of the land and the turbines:

\[
\frac{\text{Cost}_{\text{total}}}{C_{P,\text{total}}} = \left( \frac{\text{Cost}_{\text{Land}} \times A_L + \Gamma_A \times A_L \times \text{Cost}_{\text{Turbine}}}{C_{P,\text{total}}} \right)
\]

\[
= \left( \frac{\text{Cost}_{\text{Land}}}{\text{Cost}_{\text{Turbine}}} + \Gamma_A \right) \times \frac{A_L \times \text{Cost}_{\text{Turbine}}}{C_{P,\text{total}}}
\]

where $\Gamma_A$ is the wind turbine density per unit area and $A_L$ is the total lot land area. $\text{Cost}_{\text{Land}}$ is the cost of land per unit area and $\text{Cost}_{\text{Turbine}}$ is the cost of a single turbine. Using different land cost to turbine cost ratios, and the cost for a typical individual turbine similar to the one we simulated ($\approx$ $10,000$ US dollars) (Dabiri, 2011) in Equation 9, the total cost over total $C_p$ ratios were computed and plotted in Fig. 15. Using this comparison metric also indicates that the triangular cluster staggered layout has the lowest capital cost per projected unit power generated, and is therefore the best design among the ones we investigate here.
Similar analysis has been made using total $C_P^*$ and the results confirmed that wind farms with cluster designs are still the most optimal effective the one investigated here. Again, we reiterate that the comparison with $C_P$ is more relevant for small farms, while if one uses $C_P^*$ the results are more representative of large farms.

4 Conclusions

This paper presents a novel concept for optimizing the layout of large VAWT wind farms. It takes advantage of synergistic interactions between closely spaced turbines that were previously shown to yield higher power for a limited number of turbines. Using an ALM-LES model, the modelled wake generated by two-counter rotating turbines is first successfully validated against results from field experiments. To take advantage of the high wind velocity created by the flow deflection of VAWTs when placed in close vicinity, we proposed a triangular cluster design.
consisting of 3 VAWTs, which can form the basis for larger wind farms. The triangular design is the one that best exploits flow acceleration, with a minimal increase in wake shadowing.

The influence of inter-turbine spacing relative to their diameter $L/D$ was then investigated to optimize a single cluster in terms of the total generated power, the omni-directionality of its performance, and the needed downstream wake recovery distance. Changing the turbine spacing, the cases with $L/D$ ratio of 3, 4 and 5 were shown to generate the highest cluster-averaged power. Further tests were then performed with these three spacings only, and the case with $L/D$ of 5 emerged as the one with the highest cluster-averaged power over all wind directions: the generated power averaged over all wind direction for this case is about 10% higher than the power generated by three isolated turbines. Furthermore, $L/D = 5$ results in the lowest variation of the generated power with wind direction, and the downstream wake recovery distance is shorter (since the cluster is more “porous”). Therefore, this cluster design confirmed the potential to use synergistic VAWT interactions to increase power production, and would generate a higher power density (power generated per unit land used) due to the proximity of the rotors. It was hence adopted for configuring large VAWT wind farms.

Farms that use this advanced cluster design and a sufficient distance for wake recovery between clusters were then compared to prototypical aligned and staggered designs for infinitely large wind farms, with different turbine horizontal spacings of 5D, 10D and 20D. For the very large wind farms simulated, the results show that the average wind farm $C_P$ for the case with staggered triangle clusters was much higher than for the wind farms with regular configurations, reaching almost double the value of the prototypical designs for 20D turbine or cluster spacing. Using these average $C_P$ results and a simple capital cost function for the whole wind farm and varying
the land to turbine cost ratio, we also showed that the wind farm design with staggered triangle
clusters is the optimal design (amongst the ones considered here) in terms of cost per unit power
produced. This normalized cost is almost reduced in half compared to prototypical
configurations when in areas with relatively high land costs.

These results strongly indicate that VAWT farms can and should be configured using different
approaches than the ones used for horizontal axis rotors (although the potential benefits of
HAWT clustering could also be investigated). A significant increase in power and decrease in
capital costs can be achieved using the ability of VAWTs to positively boost the power
production of nearby turbines if properly configured. A further interesting aspect of the results is
that, in addition to turbine interactions within a cluster, the clusters themselves seem to interact
synergistically, further boosting power production. It should also be mentioned that one of the
criteria in optimizing the clusters and farms in this paper was omni-directionality. We sought to
propose configurations the performance of which is not strongly dependent on wind direction
since this is also a major advantage of individual VAWTs. However, if this criterion is relaxed,
e.g. in places where there is a dominant wind direction, the optimal cluster designs can be very
different and can use this synergistic interaction between clusters as well, with potentially much
higher power densities.

Finally, one important factor that plays an important role in modulating power output from a
large wind farm is atmospheric stability. Static stability in the ABL, influenced by vertical
thermal gradients, has a significant effect on flow and turbulence. It has been shown that diurnal
cycle and different ABL stabilities strongly influence the performance of large HAWT wind
farms (Abkar et al., 2016; Calaf et al., 2011), and the same is expected for our VAWT farms.
However, due to paper length constraints, this study focused on the basic neutral case only, and ALM-LES investigations of the effect of atmospheric stability on VAWT wind farms are left for future studies.

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Reference


