Energy exchanges

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Lecture 3
Back to Urban

It turns out that this new term: \(-\frac{\rho'}{\bar{\rho}} g\)

has major significance for urban microclimates, and many environmental flows. Recall, small deviations \(\rho'\) from \(\bar{\rho}\), are often due to heating/cooling of air, mainly at the surface. Let's now study the heat conservation equation to understand this.

How do the heat and momentum equation couple?
How and where are the heating/cooling bouyant forces generated?
What do they imply for the urban environment?
... in the next few lectures
Buoyancy effect on atmospheric flow

Surface heating $\rightarrow$ Unstable daytime atmosphere
Buoyancy effect on atmospheric flow

Surface cooling → Stable nighttime atmosphere

Net energy loss cools the ground

Outgoing radiation energy

Colder air resists rising
Imagine a very thin layer at the surface of the earth, a building, that is too thin to have a large thermal storage capacity (relative to the fluxes occurring through it)

\[ R_n = H + LE + G \]
\[ R_n = S_{down} + L_{down} - S_{up} - L_{up} \]
Surface Energy Budget

- $G$: the heat flux conducted into the surface (buildings, ground, etc; storage and subsequent release of heat in the ground or building canopy are very important)
- $R_n$: net radiation
- $S_{down}$: the incoming/downward direct and diffuse shortwave radiation
- $L_{down}$: the downward longwave radiation
- $S_{up} = \alpha_s S_{down}$: the reflected shortwave radiation
  $\alpha_s$ is the surface albedo for shortwave radiation
- $L_{up} = \alpha_L L_{down} + \varepsilon \sigma T_s^4$: the longwave radiation reflected & emitted by the surface
  $\alpha_L$ is the surface albedo for longwave radiation (0-1)
  $\varepsilon$ is the surface emissivity (0 - 1), and
  $\sigma$ is the Stephan-Boltzman constant $=5.67 \times 10^{-8}$ (Wm$^{-2}$K$^{-4}$)
- $H$: the sensible convective heat flux
- $LE$: the latent convective heat flux (through evaporation or condensation)
G: heat flux into the surface by conduction

- Transfer of heat (molecular KE) from one molecule to another
  - Energy travels from hot to cold
- In solids, by vibration of molecules or moving free electrons
- In fluid and gases, by collision and diffusion of molecules, without bulk flow
Fourier's law (as with diffusion)

in 1D: \[ q = -k \frac{dT}{dx} \]

general form: \[ \overrightarrow{q} = -k \nabla T \]

where \( k \) is the thermal conductivity (Wm\(^{-1}\)K\(^{-1}\))

Again, as with diffusion, heat_in – heat_out:

\[ \frac{\partial T}{\partial t} = \frac{k}{\rho c} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \]
R_n: net radiation

= sum of 4 radiation components

- Energy can travel through matter or vacuum as electromagnetic waves → Radiation
- Any matter with \( T > 0 \) K radiates energy
- An electromagnetic wave is a coupling of an electric wave and a magnetic wave that are perpendicular to each other and perpendicular to the propagation direction of the EM wave
- All 3 waves have therefore to have the same wavelength:

\[
\lambda = \frac{c}{\text{frequency}} \quad (c=\text{speed of light})
\]
- The frequency and energy of the radiation depend on the temperature of the emitting body
Wavelength and body temperature

- Penetrates Earth's Atmosphere?
- Radiation Type
  - Radio: $10^3$ m
  - Microwave: $10^{-2}$ m
  - Infrared: $10^{-5}$ m
  - Visible: $0.5 \times 10^{-6}$ m
  - Ultraviolet: $10^{-8}$ m
  - X-ray: $10^{-10}$ m
  - Gamma ray: $10^{-12}$ m
- Approximate Scale of Wavelength
  - Buildings
  - Humans
  - Butterflies
  - Needle Point
  - Protozoans
  - Molecules
  - Atoms
  - Atomic Nuclei
- Frequency (Hz)
  - $10^4$
  - $10^8$
  - $10^{12}$
  - $10^{15}$
  - $10^{16}$
  - $10^{18}$
  - $10^{20}$
- Temperature of objects at which this radiation is the most intense wavelength emitted
  - 1 K
  - 100 K
  - 10,000 K
  - 10,000,000 K
- Terrestrial: 1 to 100 microns
- Solar: 0.1 to 10 microns
Quantitative Description

- A **blackbody** absorbs all incoming/incident radiation and emits *all the radiation it can emit*

- Stephan-Botzeman law for blackbody radiation:

  \[ F = \text{radiation emitted from a black body (Wm}^{-2} \text{)} = \sigma T^4 \]
  \[ \sigma = \text{Boltzman Constant} = 5.67 \times 10^{-8} \text{ (Wm}^{-2} \text{K}^{-4} \text{)} \]
  \[ T = \text{Body temperature (K)} \]

- Recall that W=J/s
Quantitative Description

- A Grey Body reflects part of the incident radiation, absorbs a part, and transmits the rest
- Albedo/Reflectivity = Reflected Radiation / Incident Radiation
- Absorptivity = Absorbed Radiation / Incident Radiation
- Transmissivity = Radiation Passing Through / Incident Radiation

Absorptivity + Transmissivity + Albedo = 1

- It also emits a fraction of the radiation it would emit if it was a black body
  \[ F_{\text{grey}} = \varepsilon \sigma T^4 \]

  Kirchhoff’s law  \( \varepsilon = \text{emissivity of the grey body (units?)} \)

  \( \varepsilon^* = \text{absorptivity at thermal equilibrium and more} \)

- Hence if transmissivity = 0 (soil, wall)

\[ \rightarrow \text{emissivity} = 1 - \text{albedo} \rightarrow \varepsilon = 1 - \alpha \]

Used often to relate the emissivity and long wave albedo of surfaces
The greenhouse effect

Carbon dioxide  Methane
Nitrogen Oxide  Chlorofluorocarbons
Global average of the SEB

What is G on average?

\[ R_n = H + LE + G \]

\[ R_n = S_{down} + L_{down} - S_{up} - L_{up} \]

\[ = (32 + 23) + 96 - 4 - 117 = 30 \]
Spatial Variation of $R_n$

\[ R_n = H + LE + G < 0 \]
\[ R_n = H + LE + G > 0 \]
\[ R_n = H + LE + G < 0 \]

This imbalance is the main driver of the oceans and atmospheres.
Diurnal Variation of $R_n$

- High $R_n$: clear summer days
- Low $R_n$: cloudy fall days
4-component radiometers
Radiation on Princeton roof

August 10, 2009

[Graph showing radiation levels over time with different lines for SW down, SW up, LW down, LW up, and Net.]
Urban radiation

- Roofs: single absorption
- Radiative trapping: multiple reflection and absorptions
- Asphalt: strong absorption
- Radiative trapping

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Albedo of natural surfaces (Brutsaert, 2005)

<table>
<thead>
<tr>
<th>Nature of surface</th>
<th>Albedo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deep water</td>
<td>0.04–0.08</td>
</tr>
<tr>
<td>Moist dark soils; ploughed fields</td>
<td>0.05–0.15</td>
</tr>
<tr>
<td>Gray soils, bare fields</td>
<td>0.15–0.25</td>
</tr>
<tr>
<td>Dry soils, desert</td>
<td>0.20–0.35</td>
</tr>
<tr>
<td>White sand; lime</td>
<td>0.30–0.40</td>
</tr>
<tr>
<td>Green grass and other short vegetation (e.g. alfalfa, potatoes, beets)</td>
<td>0.15–0.25</td>
</tr>
<tr>
<td>Dry grass; stubble</td>
<td>0.15–0.20</td>
</tr>
<tr>
<td>Dry prairie and savannah</td>
<td>0.20–0.30</td>
</tr>
<tr>
<td>Coniferous forest</td>
<td>0.10–0.15</td>
</tr>
<tr>
<td>Deciduous forest</td>
<td>0.15–0.25</td>
</tr>
<tr>
<td>Forest with melting snow</td>
<td>0.20–0.30</td>
</tr>
<tr>
<td>Old and dirty snow cover</td>
<td>0.35–0.65</td>
</tr>
<tr>
<td>Clean, stable snow cover</td>
<td>0.60–0.75</td>
</tr>
<tr>
<td>Fresh dry snow</td>
<td>0.80–0.90</td>
</tr>
</tbody>
</table>
Albedo of engineered surfaces (< than natural)

<table>
<thead>
<tr>
<th>Material surface</th>
<th>Albedo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black acrylic paint</td>
<td>0.05 Solar Reflectance</td>
</tr>
<tr>
<td>New asphalt</td>
<td>0.05</td>
</tr>
<tr>
<td>Aged asphalt</td>
<td>0.1</td>
</tr>
<tr>
<td>“White” asphalt shingle</td>
<td>0.2</td>
</tr>
<tr>
<td>Aged concrete</td>
<td>0.2 to 0.3</td>
</tr>
<tr>
<td>New concrete (traditional)</td>
<td>0.4 to 0.5</td>
</tr>
<tr>
<td>New concrete with white portland cement</td>
<td>0.7 to 0.8</td>
</tr>
<tr>
<td>White acrylic paint</td>
<td>0.8</td>
</tr>
</tbody>
</table>
Albedos and surfaces in an urban area

- White paint: 0.50 - 0.90
- Red/brown tile: 0.10 - 0.35
- Grass: 0.25 - 0.30
- Trees: 0.15 - 0.18
- Tar and gravel roof: 0.03 - 0.18
- Brick/stone: 0.20 - 0.40
- Concrete: 0.10 - 0.35
- Corrugated roof: 0.10 - 0.15
- Asphalt: 0.05 - 0.20
Urban heat storage

- Storage of heat during the day
- Heat release at night

Urban Canopy

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Thermal properties of common urban materials

<table>
<thead>
<tr>
<th>Material</th>
<th>$k = $ Thermal conductivity (W/ m.K)</th>
<th>$k/\rho c_p = $ Thermal diffusivity (cm$^2$/s)</th>
<th>$\rho c_p$ (J/Km$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement</td>
<td>0.3</td>
<td>0.0013</td>
<td>$2.31 \times 10^6$</td>
</tr>
<tr>
<td>Concrete</td>
<td>1.28</td>
<td>0.0066</td>
<td>$1.94 \times 10^6$</td>
</tr>
<tr>
<td>Asphalt</td>
<td>0.7</td>
<td>0.0036</td>
<td>$1.94 \times 10^6$</td>
</tr>
<tr>
<td>Brick</td>
<td>0.38-0.52</td>
<td>0.0028-0.0034</td>
<td>$\sim 1.53 \times 10^6$</td>
</tr>
<tr>
<td>Limestone</td>
<td>1.1</td>
<td>0.005</td>
<td>$2.2 \times 10^6$</td>
</tr>
<tr>
<td>Snow (temp &lt; 0ºC)</td>
<td>0.05 - 0.25</td>
<td>0.0039</td>
<td>$\sim 0.38 \times 10^5$</td>
</tr>
<tr>
<td>Wood</td>
<td>0.17</td>
<td>0.0012</td>
<td>$1.42 \times 10^6$</td>
</tr>
<tr>
<td>Dry Soil</td>
<td>1</td>
<td>0.004</td>
<td>$2.5 \times 10^6$</td>
</tr>
</tbody>
</table>

- Concrete: conducts and stores a lot of heat, stays hot later into the night
- Asphalt: can store as much but conducts less, but low albedo → hot T, absorbs/stores a lot near the surface
- Wood: great for insulation
Surface temperatures over Princeton (model and measurements, more later)

- Asphalt much hotter than concrete or vegetation (low albedo)
- Concrete cycle is slightly delayed (high conductivity and storage)
- Grass is cooler than concrete despite lower albedo (evaporation)
Advection/convection of heat and water vapor & atmospheric stability

Turbulent fluxes from the surface modify atmospheric stability
Which is lighter?

$PU$
Moisture and stability

Which is lighter?

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Virtual temperature $T_v$

- A moist parcel of air is lighter than a dry parcel of air with the same volume (water vapor is lighter than air)
- As such, if the moist parcel loses all its water vapor molecules and replaces them with dry air molecules, its density will increase.
- To regain/maintain its initial density it has to heat up.
- How “hotter” depends on how humid the moist parcel was.
- The virtual temperature of a moist parcel is the temperature it should reach to keep the same density if it were to lose all its vapor (replaced with air) and become dry.
- It is a corrected temperature into which we also lump the effect of moisture content on density.

For example, if we want to compare the densities of a moist and a dry (or drier) parcel of air, we can simply compare their $T_v$. 

Slide 29
Virtual temperature $T_v$

$T_v = (1+0.61q) T$ \hspace{1cm} \text{Proof as exercise}

$q = \rho_{WV}/\rho$ is the specific humidity \hspace{1cm} (kg$_{WV}$/kg$_{MA}$)

$\rho_{WV} =$ density of water vapor (WV) alone \hspace{1cm} (kg$_{WV}$/m$^3$)

$\rho_{DA} =$ density of dry air alone (DA) \hspace{1cm} (kg$_{DA}$/m$^3$)

$\rho = \rho_{WV} + \rho_{DA}$ density of mixture \hspace{1cm} (kg$_{DA}$/m$^3$)

$m =$ mixing ratio $= \rho_{WV}/\rho_{DA}$

$RH = m/m^* \equiv e/e^*$

where $e$ is the vapor pressure, and $^*$ denotes the value at saturation

$\rho = \frac{p}{RT} = \frac{p}{R_d T_v}$

$R =$ gas constant for moist air \hspace{0.5cm} , \hspace{0.5cm} R_d =$ gas constant for dry air
Height and stability: Potential Temperature $\theta$

- Imagine 2 dry air parcels on top of each other in a static atmosphere.
- They have the same density, but different pressures and temperatures.
  \[ \rho = \frac{P}{R_d T} \]
- The lower one must have a higher $T$ since it is at a higher $P$ (hydrostatic).
- If we only compare temperatures we would be misled into thinking the lower parcel should rise since it is hotter than the parcel above it, but it does not since they have the same density.
- In this framework, temperatures of air parcels at different height cannot be directly compared to deduce stability, without also accounting for pressure differences.
- To see if the lower parcel will rise, we should compare its temperature when it would reach the higher elevation (lower than original $T$) to the temperature at that elevation, but such a comparison would depend on initial and final heights ...
- This $T$ variation is due to $P$ variation.
Potential Temperature $\theta$

- The potential temperature removes the effect of this vertical variability by bringing all the air to a reference pressure i.e. by computing the temperature field in the atmosphere if it all were at the same pressure of 10000hPa
Hydrostatic equilibrium

\[
\frac{dp}{dz} = -\rho g
\]

\[p - p_0 = -\rho g(z - z_0)\]

with ideal gas law \(p = \rho R_d T\)

\[
\Rightarrow \frac{p}{p_0} = \left(\frac{T}{T_0}\right)^\frac{g}{R_d \Gamma}
\]

(Poisson Equation)

\(\Gamma = -dT / dz\) is the environmental lapse rate

Given the difference of \(p\) (\(T\)) between 2 levels, one can directly get the difference in \(T\) (\(p\))
Potential Temperature $\theta$

The poisson equation applies to an ideal gas in a static atmosphere,

$$\frac{p}{p_0} = \left(\frac{T}{T_0}\right)^{\frac{g}{R_d\Gamma}}$$

relates temperature and pressures at 2 elevations.

Now we can also apply it to a single parcel of air rising and falling to see how $T$ and $P$ change with height, but in this case we need to use the adiabatic lapse rate $\Gamma_d = g / c_{p,d}$ (from the first law of thermodynamics, exercise) Why?

Hence if a parcel of air moves from a level $(P,T)$ to a level $(P_0, T_0 = \theta)$

$$\frac{p}{p_0} = \left(\frac{T}{\theta}\right)^{\frac{g}{\Gamma_d R_d}} \rightarrow \theta = T\left(\frac{p_0}{p}\right)^{\frac{\Gamma_d R_d}{g}} \rightarrow \theta = T\left(\frac{p_0}{p}\right)^{\frac{R_d}{c_{p,d}}} \quad \text{AND} \quad \theta_v \approx T_v\left(\frac{p_0}{p}\right)^{\frac{R_d}{c_{p,d}}}
$$

$\theta_v =$ potential virtual temperature $\approx$ virtual potential temperature
Lapse rates in the atmosphere

- Dry Adiabatic Lapse Rate = cooling of a parcel of air that rises adiabatically and adjusts to the ambient Pressure.

- Wet or Saturated or Moist Adiabatic Lapse Rate = cooling of a parcel of air that rises adiabatically and adjusts to the ambient Pressure while its moisture is condensing.

- Dry and Wet adiabatic Lapse rates: “almost” independent of the environmental lapse rate and tell us little about the atmosphere and its state.
Lapse rates in the atmosphere

- Dry Adiabatic Lapse Rate = \(-dT/dz = \Gamma_d = g/c_{p,d} = 9.8 \, ^\circ K/km\)

- Wet or Saturated or Moist Adiabatic Lapse Rate
  \[ = -dT/dz = \Gamma_s = g/c_{p,d} + (L_e / c_p) (dq/dz) \]
  \[ \approx 5.5 \, ^\circ K/km \text{ on average but depends on } dq/dz \]
Vertical Stability of the atmosphere

- Actual Lapse Rate = \(- \frac{dT}{dz}\) as measured in the atmosphere

\[ \Gamma > \Gamma_d \]

Neutral \( \Gamma = \Gamma_d \)

\[ \Gamma < \Gamma_d \]
Vertical Stability of the atmosphere

- Actual Lapse Rate = $-\frac{dT}{dz}$ as measured in the atmosphere
Vertical Stability of the atmosphere: why $\theta_v$?

- Dry adiabatic lapse rate of $\theta_v = 0$
- Moist adiabatic lapse rate of $\theta_v$
- Conditional Stability
- Absolute Stability
- Absolute Instability
why $\theta_v$?

- Inversely proportional to density

$$\rho = \frac{p}{RT} = \frac{p}{R_d T_v} = \frac{p}{R_d \theta_v} = \frac{p_0^{(1-R_d/c_{p,d})} R_d/c_{p,d}}{R_d \theta_v} = \text{cst}$$

$$\Rightarrow \frac{\theta_v'}{\theta_v} \approx -\frac{\rho'}{\rho}$$

(for a given parcel at an initially defined $p$)

- $\theta_v$ and $\theta$ conserved, even when work changes internal energy (vertical motion), rewrite energy conservation in terms of $\theta$, which only changes due to heat exchanges rather than work

$$\frac{\partial \theta}{\partial t} + \vec{u}.\nabla \theta = \alpha \nabla^2 \theta - \frac{1}{\rho c_p} \left( \nabla \cdot \vec{R} \right) - \frac{L_e}{\rho c_p} E$$

Modify momentum equation
Static Stability of the Troposphere

On average, the troposphere is **statically stable** with a potential temperature gradient of about 3.3 K/km → Surface mixing → inversion
Troposphere is statically stable, but daytime surface heating creates an unstable lower layer: the atmospheric boundary layer.
What characterizes ABL flow?

- Highly turbulent (high Reynolds number)
- Not strongly affected by rotation of the earth Coriolis

\[
\frac{\text{Inertial Force}}{\text{Coriolis Force}} = \frac{\vec{u} \cdot \nabla \vec{u}}{2\Omega \times \vec{u}} \sim \frac{UU}{\Omega U} = \frac{U}{\Omega L} = \text{Rossby Number} = Ro \gg \text{in ABL}
\]

*U*: characteristic velocity of the flow ~ 10m/s in the ABL

*L*: characteristic length of the flow ~ 1000m in the ABL

Actual height can range from:
100 m (or even less) up to 4 km
Recommended readings

- Roland Stull: An Introduction to Boundary Layer Meteorology
- Adrian Bejan: Heat Transfer
- Wilfried Brutsaert: Hydrology: an Introduction